Testing Low Energy Theorems in Nucleon-Nucleon Scattering

Thomas D. Cohen

Department of Physics, University of Maryland, College Park, MD 20742-4111

James M. Hansen

Montgomery Blair High School, Silver Spring, MD 20901

Low energy theorems have been derived for the coefficients of the effective range expansion in s-wave nucleon-nucleon scattering valid to leading nontrivial order in an expansion based Q counting, a scheme in which both m_π and $1/a$ (where a is the scattering length) are treated as small mass scales. Previous tests of these theorems based on coefficients extracted from scattering data indicate a pattern of gross violations which suggested serious problems for the perturbative treatment of pions implicit in Q counting. We discuss the possibility that uncertainties associated with extracting the coefficients from the scattering data make such tests invalid. Here we show that errors in the s-wave phase shift extractions are sufficiently small to test direct test predictions from Q counting at next to leading order. In particular we show that there exist low energy theorems for the sum of all terms in the effective range expansion beyond the first two which allow for precise tests. These low energy theorems fail badly which suggests that pionic aspects of Q counting are not under control.

I. INTRODUCTION

There has been considerable interest in the use of effective field theory (EFT) techniques in nuclear physics during the past several years [1–30]. Much of the goal of this work is to use power counting ideas associated with chiral symmetry to nuclear physics. This is not simple since apart from m_π, the inverse s-wave scattering length, $1/a$ is another light scale in the problem. Many of the approaches beginning with Weinberg's [1] formulate the expansion at the level of a two-particle irreducible kernel rather than for observables. While such an approach provides an organizing principle for calculations, it provides no systematic estimate of the accuracy of particular observables in terms of power counting. Recently a scheme was introduced in which observables can be expressed in terms of a consistent power counting scheme [21,26–29].

This scheme is based on power counting in a single scale, Q

$$m_\pi \sim Q \quad 1/a \sim Q \quad k \sim Q \tag{1}$$

In this power counting, all other scales are assumed to be heavy and will collectively be symbolized by Λ. This power counting scheme describes low momentum physics in that $k/\Lambda \ll 1$. There can be rapid momentum dependence of some observables, however, since the expansion for any observable includes all orders in ka and k/m_π. This power counting scheme has been implemented using dimensional regularization [21,28,29] and directly in configuration space using a cutoff [26]. We note in passing the fact that $1/a$ is formally treated as being of the same order as m_π and k is not emphasized in the original papers of Kaplan, Savage and Wise [21,22]. It is implicit, however, in the expression for the leading order (Q^{-1}) amplitude which is given by $-4\pi/[M\,(1/a + ik)]$. Note that if k and $1/a$ were of different orders one could expand out the denominators. One test that the Q counting formally involves treating $1/a$ as being of the same order is found in the cutoff treatment of ref. [26] where the rules in eq. (1) were explicitly used to derive an expression for the phase shifts which is formally equivalent to the expressions derived by Kaplan, Savage and Wise [21,22].

In a previous paper [27] we used Q counting to derive low energy theorems for coefficients of the effective range expansion (ERE) at leading nontrivial order in Q counting. The ERE is a parameterization of s-wave scattering given by

$$k\cot(\delta) \;=\; -\frac{1}{a} \,+\, \frac{1}{2} r_e\, k^2 \,+\, v_2\, k^4 + \, v_3\, k^6 + \, v_4\, k^8 + \ldots \tag{2}$$

and is particularly useful in the case of unnaturally large a. The v_i coefficients at this order are fixed entirely by m_π and $1/a$. The low energy theorems were compared with v_i extracted from a partial wave analysis of the scattering data. All of the predictions were many times larger than the v_i extracted from scattering data. As the low energy theorems are particularly sensitive to pion physics (all terms are nonanalytic in m_π) a plausible conclusion from this discrepancy is that the part of Q counting associated with the expansion of m_π/Λ has broken down. Such a conclusion is consistent with the many successes of Q counting [22] for deuteron properties provided such successes depend essentially on the expansion of $1/(a\Lambda)$ rather than m_π/Λ. Indeed, in ref. [22] the authors show that the the effective range expansion without explicit pions does a better job of describing the form factors at low momentum transfers than the theory based on Q counting with explicit pions. This is precisely what one would expect if the $1/(a\Lambda)$ expansion were working and the m_π/Λ failing.

The scenario where Λ is numerically of the same scale as m_π is quite plausible. The essence of Q counting is that the *only* long distance scales are a and $1/m_\pi$. As a practical matter one should identify $1/\Lambda$ as the longest of the various short distance scales in the problem as that will be the scale responsible for a breakdown of the expansion. The effective range, r_e, is an important scale characterizing low energy nucleon-nucleon scattering. Numerically it is ~ 2.7 fm in the singlet channel and ~ 1.7 fm in the triplet channel. In both cases, $r_e\, m_\pi > 1$. If one identifies $1/r_e$ as a short distance scale, Λ, then $m_\pi/\Lambda > 1$ and a chiral expansion is not valid. Two issues must be resolved before coming to such a conclusion. The first is whether r_e is a "short distance" scale (which just happens to be numerically long), and the second is whether the appropriate scale

is $1/r_e$, or $1/r_e$ times some numerical factor which if large enough might render the chiral expansion useful.

The first issue can be easily resolved in the context of Q counting. If in Q counting, r_e were of order Q^{-1}, for example, scaling as a or $1/m_\pi$, then the large numerical value of r_e would be natural. However, the effective range has been calculated at leading nontrivial order in Q counting (*i.e.,* next to leading order) [29] and it is explicitly seen that $r_e \sim Q^0$. Thus, the value of r_e in the context of Q counting is determined by short distance scales. This in turn suggests that the longest scale treated as short distance in Q counting (namely r_e) is comparable to or larger than the shortest longest scale ($1/m_\pi$). The fact that $r_e m_\pi \geq 1$ suggests that the chiral expansion may not be under control even when the unnaturally large scattering length is taken into account.

The issue of whether the large value of the effective range invalidates the chiral aspects of Q counting is central to the effective field theory program in nuclear physics. The question of whether the low energy theorems of ref. ([27]) are badly violated is, in turn, a critical issue in assessing the viability of the chiral aspects of the Q counting scheme. Recently, Mehen and Stewart [28] have raised the question of whether errors in the phase shifts render a reliable extraction of the v_i coefficients impossible. They make a crude estimate of the errors of the v_2 in the triplet channel coefficient including the uncertainties using the reported values from the Nijmegen partial wave analysis for $k < 70$ MeV along with the scattering length and effective range and obtain $v_2 = -.50 \pm .52 \pm \sim .1\,\mathrm{fm}^3$, where the first error is a quadrature sum of the estimates errors and the second uncertainty is a theoretical estimate of the contributions from the v_3 and higher terms. This estimate is consistent with both the value fit from the Nijmegen analysis [32], $v_2^{\rm fit} = .04$ fm^3, and the low energy theorem prediction value of $v_2^{\rm LET} = .95$ fm^3. Using the second lowest report point from the Nijmegen analysis they estimate the $v_2 = .03 \pm .04 \pm \sim .5\,\mathrm{fm}^3$. Accordingly they conclude that there is too much uncertainty in the extraction of the v_i coefficients to make a sharp test of the low energy theorems.

In this paper we will show that data are sufficiently good so that sharp tests of the low energy theorems of ref. [27] are possible and that the theorems are, in fact, badly violated. The most sensitive method is to consider weighted sums of the low energy theorems of the v_i coefficients which can be extracted with far greater than precision than the individual terms. In particular, we test the total contribution to $k \cot\delta$ arising from all of the higher terms (v_2 and above) in the effective range expansion. In this paper we will focus on tests in the triplet channel. One particularly nice place to test is at the deuteron pole ($k = i\sqrt{MB}$ where B is the binding energy) which is known with great precision. One can also work at small real k and compare with the uncertainties in the partial wave analysis. In this analysis, we find in every case that the Q counting at second order makes predictions which are incompatible with the data.

II. TESTS OF THE LOW ENERGY THEOREMS

For the following analysis it is useful to write an expression for $k\cot(\delta)$ excluding the contributions from the scattering length and effective range terms. We will refer to this quantity as the shape function and

denote it, $\mathcal{S}(k^2)$. Thus,

$$\mathcal{S}(k^2) \equiv k\cot(\delta) - \left(-\frac{1}{a} + \frac{1}{2} r_e k^2\right) \tag{3}$$

$$= \sum_{j\geq 2} v_j k^{2j} \tag{4}$$

where the second equality holds only within the radius of convergence of the effective range expansion. Note, however, that the general definition holds for all k^2. The shape function, $\mathcal{S}(k^2)$, can be calculated in the Q expansion. Using the expression for $k\cot(\delta)$ in ref. [27] which can be obtained using either a cutoff scheme or dimensional regularization with either PDS or OS subtraction, one finds at order Q^2 for either the singlet or triplet channel

$$\begin{aligned}\mathcal{S}^{\mathrm{LET}}(k^2) \;=\; & \frac{g_A^2 M}{16\pi^2 f_\pi}\left[\frac{1}{a^2} - \frac{2m_\pi}{a} + k^2\left(-1 + \frac{8}{3m_\pi a} - \frac{2}{m_\pi^2 a^2}\right)\right] \\ & - \frac{1}{a^2}\frac{g_A^2 M}{64\pi f_\pi^2}\left(\frac{m_\pi^2}{k^2}\right)\ln\left(1 + \frac{4k^2}{m_\pi^2}\right) \\ & + \frac{m_\pi}{a}\frac{g_A^2 M}{16\pi f_\pi^2}\left(\frac{m_\pi}{k}\right)\tan^{-1}\left(\frac{2k}{m_\pi}\right) + m_\pi^2\frac{g_A^2 M}{64\pi f_\pi^2}\ln\left(1 + \frac{4k^2}{m_\pi^2}\right) + \mathcal{O}(Q^3)\end{aligned} \tag{5}$$

The predicted v_i coefficients predicted at this order in the Q expansion are obtained by differentiating the preceding expression with respect to k,

$$v_j = \frac{1}{(j-2)!}\left.\frac{\partial^2 \mathcal{S}^{\mathrm{LET}}}{\partial k^2}\right|_{k=0} \tag{6}$$

This gives

$$\begin{aligned} v_2 &= \frac{g_A^2 M}{16\pi f_\pi^2}\left(-\frac{16}{3a^2\, m_\pi^4} + \frac{32}{5a\, m_\pi^3} - \frac{2}{m_\pi^2}\right) \\ v_3 &= \frac{g_A^2 M}{16\pi f_\pi^2}\left(\frac{16}{a^2\, m_\pi^6} - \frac{128}{7a\, m_\pi^5} + \frac{16}{3m_\pi^4}\right) \\ v_4 &= \frac{g_A^2 M}{16\pi f_\pi^2}\left(-\frac{256}{5a^2\, m_\pi^8} + \frac{512}{9a\, m_\pi^7} - \frac{16}{m_\pi^6}\right) \\ & \ldots \end{aligned} \tag{7}$$

The effective range expansion from eq. (5) has a finite radius of convergence. The existence of a cut at $k^2 = -m_\pi^2/4$ implies that the series only converges for $k^2 < m_\pi^2/4 \approx 70$MeV. However this limitation on region of the validity of the effective range expansion is *not* a limitation on the range of validity of eq. (5). It is valid up to corrections of order Q/Λ even for $k^2 > m_\pi/2$. Indeed, the entire motivation underlying the development of the Q expansion was a scheme valid when $k \sim m_\pi$ [21].

Note that the prediction of the shape function $\mathcal{S}(k^2)$ depends on no free parameters and thus is a low energy theorem in the same sense that the predictions for the v_i's are low energy theorems. The low energy theorem for $\mathcal{S}(k)$ is more basic than the low energy theorems for the v_i; all the predicted v_i follow from eq. (5). It is also important to note that $\mathcal{S}(k^2)$ at fixed k^2 is far easier to extract from the data with reliable error estimates than v_i since all that is needed to be known is the phase shift, the scattering length and effective range, along with knowledge of their errors. One does not need to know enough information to accurately deduce higher derivatives of the function. It should be noted, that within the radius of convergence of the effective range expansion, *i.e.* for $k^2 < m_\pi^2/4$, testing the predicted $\mathcal{S}(k^2)$ tests a sum of the low energy theorems for the v_j weighted by k^{2j}. However, as noted above there is no necessity to restrict tests of $\mathcal{S}(k^2)$ to this regime.

It is important to note that the shape function, $\mathcal{S}(k^2)$, like the individual v_j's provide an ideal way to probe the pionic aspects of the Q counting scheme. Recall that in the Q counting scheme there are two small mass scales apart from the external momentum, $1/a$ and m_π. However, $1/a << m_\pi$. Thus, it remains possible that the underlying "short distance" scale, Λ, is in fact comparable to m_π while $1/a << \Lambda$. If such a situation occurs one expects observables primarily sensitive to $1/(a\Lambda)$ to be well described, whereas observables primarily sensitive to m_π/Λ to be poorly described. We note that this possibility is not implausible given experience with potential models which are fit to the data where it is generally seen that the non-one-pion-exchange part of the potential remains significant at ranges comparable to $1/m_\pi$ so that there is a "short distance" scale in the problem of the pionic range [26] . The most straightforward way to test whether the m_π/Λ expansion is under control is to compare predictions from a theory with pions integrated out to those which include pions and see whether one gets systematic improvement by including the pions. Unfortunately for generic observables this test is not very clean since the observable may be completely dominated by the $1/(a\Lambda)$ expansion. On the other hand, if one has an observable which vanishes at some order in the pion-integrated out theory but not in the pion-included theory than one has a prediction which explicitly tests the pionic contributions. The shape function, $\mathcal{S}(k^2)$, at order Q^2 is such an example (as are the v_i coefficients derived from it). The reason for this is that $k \cot \delta$ in the pion-integrated-out theory is just the effective range expansion, which at order Q^2 truncates at the second term and implies that $\mathcal{S}(k^2) = 0$ at this order. Thus the predictions of $\mathcal{S}(k^2)$ provides a sharp test of the pionic part of Q counting.

In this paper we will restrict our attention to the triplet channel as in that channel a and r_e have been extracted from the partial wave analysis with very small error bars [34] allowing for a very sharp test. They are given by

$$a \;=\; 5.420 \pm .001\,\text{fm} \quad r_e \;=\; 1.753 \pm .002\text{fm} \tag{8}$$

An additional advantage to working in the triplet channel is the existence of the deuteron bound state which corresponds to a pole in the scattering amplitude when it is analytically continued to imaginary momentum. Define the quantity, γ, as

$$\gamma = \sqrt{MB} \tag{9}$$

where B, the deuteron binding energy, is known with great precision to be B = 2.224575(9). The pole occurs at $k = i\gamma$ and is fixed by the condition that denominator of the scattering amplitude vanishes. This in turn fixes the value of our shape function at $k^2 = -\gamma^2$:

$$\mathcal{S}(-\gamma^2) = \frac{1}{a} + \frac{1}{2} r_e \gamma^2 - \gamma = -.017 \pm .012\text{MeV} \tag{10}$$

In contrast, the low energy theorem gives $-.743$ MeV which deviates from the extracted value by more than 6 σ. It is hard to argue that the discrepancy can be attributed to uncertainties in the data.

One can also test the low energy theorem for $\mathcal{S}(k^2)$ for real k. We have used the values in the Nijmegen phase shift analysis [32]. The extraction of $\mathcal{S}(k^2)$ from the data involves subtracting the first two terms of the effective range expansion from the extracted $k\cot(\delta)$. Since both of these quantities are intrinsically much larger than $\mathcal{S}(k^2)$, it is essential for both quantities to be given with as much precision as possible. In particular, one must be careful to use the relation between lab energy and center of mass momentum from ref. [32] which includes relativistic effects and the proton-neutron mass difference. In table I we compare the extracted value of the shape function, $\mathcal{S}(k^2)$, with the predictions from the low energy theorems. We have decided to include in this comparison lab energies up to 50 MeV, corresponding to a momentum of 153 MeV, which is approximately m_π. As noted earlier, this highest energy is outside the radius of convergence of the effective range expansion ($k = m_\pi/2$). This is not a concern if $\Lambda >> m_\pi$ (the assumption underlying the chiral part of the Q expansion), since under this assumption $k \sim m_\pi$ is within the presumed domain of validity of the Q^2 expansion. (We note that the points at the deuteron pole, and at positive energies of 1 and 5 MeV are within the radius of convergence of the effective range expansion.) In all cases except for $\mathrm{T_{lab}} = 1$ MeV, we find that the low energy theorem predicts $\mathcal{S}$ significantly greater in magnitude and the opposite sign from the extracted value. (For $\mathrm{T_{lab}} = 1$ MeV, the low energy theorem presumably disagrees with the sign of the actual value of $\mathcal{S}$ but the sign of the extracted value is undetermined since the value is consistent with zero.) The uncertainties associated with the extraction are also given in the table. The significant point is that for all cases the disagreement between the extracted value and the low energy theorem prediction is many standard deviations, even for the smallest values of k. The statistical significance of the discrepancy grows with k so that by the top of our energy range the predicted value differs from the extracted one by more than 100 σ. Clearly, the data has sufficient precision to test the low energy theorem and it is equally clear that the low energy theorem fails to correctly predict $\mathcal{S}$ even up to the sign.

The argument given above demonstrates conclusively that the Q counting scheme at second order fails to predict the shape function $\mathcal{S}(q^2)$. The central purpose of this paper is to demonstrate explicitly that there exist observables which are dominated by pionic physics for which Q counting fails. For this purpose the results discussed above is quite sufficient. It is nevertheless of some interest to ask whether the data is good enough to test the low energy theorems for the individual v_j coefficients in the effective range expansion or

whether, as suggested in ref. [28], the uncertainties are too large. The difficulty of accurately extracting high derivatives of functions from data with uncertainties is clear.

Fortunately, there are effectively several distinct fits to the scattering data. Note that the Nijmegen group not only fit the data directly in their partial wave analysis [32], they also fit several potential models directly to the data—*i.e., not the the partial wave analysis phase shifts* with a χ^2 per degree of freedom of 1.03, essentially unity [33]. In effect, as noted in ref. [33], the phase shifts produced by these potential models represent parameterizations of the partial wave analysis phase shifts. Now, as discussed in ref. [34] the effective range expansion coefficients v_j extracted from the various potential models agree with each other and with the v_i extracted directly from the best fit values of the partial wave analysis to an extremely high precision. This is quite useful, since the bias introduced is presumably quite different in the the various fits. Thus, overall the spread between the various potential models and the direct fit to the partial wave analysis should provide some sense of the scale of the uncertainty.

In table II we reproduce the triplet channel v_j extracted from the partial wave analysis fits and from the potential models and the values from the low energy theorems. Note that for all cases the spread between the different extracted values is quite small. The largest relative spread is in the v_2 coefficient values and that is presumably because v_2 is accidentally very small. In all cases, the spread in the values of the coefficients is vastly smaller than the difference from any of these values to the one predicted by the low energy theorems. This strongly suggests that the individual v_j coefficients are known well enough to test the low energy theorems and that low energy theorems make predictions inconsistent with the data.

III. DISCUSSION

By focusing on the quantity $\mathcal{S}(k^2)$, we have been able to show that at least one pion aspect of Q counting fails badly at next to leading order. ¿From our analysis it is clear that if the uncertainty estimates of ref. [32] are even approximately correct, then the predictions for $\mathcal{S}(k^2)$ from the low energy theorems are in marked disagreement with the data, even to the point of getting the sign wrong. One obvious explanation for this is the one advanced in our previous paper [27] and discussed in the introduction, namely that $1/m_\pi$ is not long ranged compared to other scales in the problem. This possibility is plausible on its face, since it is known in nuclear physics that there are many length scales which are comparable to $1/m_\pi$ but which have no obvious chiral origin. The effective range is a good example. Another example is the characteristic ranges of the non-pion-exchange part of nuclear potential which are fit to phase shifts (although as discussed in ref. [8] the need to fit the effective range constrained the non-pionic part of the potential to be long). While this does not prove that the pionic part of Q counting must fail, it certainly makes it very plausible.

If the failure of the low energy theorems for scattering indicates a systematic failure of pionic effects of s-wave properties in Q counting, one expects failure for other observables in the sense that the explicit inclusion of pions should not lead to improved predictive power. The recent calculations of deuteron form

factors in ref. [22] strongly support this view. The calculation of the form factors using a simple effective range expansion treatment including up to the effective range describes the data better than the next-to-leading order treatment including explicit pions. Had the pionic aspects of Q counting been under control one would have expected the calculation including explicit pions would have improved things.

At present we know of no observable associated with s-wave two nucleon states for which the inclusion of explicit pions in Q counting improves predictions and several for which it worsens them. Of course, this does not prove that the m_π/Λ expansion will generally fail for all s-wave observables. It remains possible, for example, that one coefficient in the next-to-leading order theory is accidentally large and that by fitting it and working at next-to-next-to leading order the usefulness of the m_π/Λ will be manifest. The authors of ref. [22] assert (without proof) that at higher orders the effective field theory with pions will work better than the simple effective range calculation since it has the correct underlying physics. We believe that this scenrio is unikely in view of the fact that there seems to be no scale separation between $1/m_\pi$ and "short distance" scales.

It is clear how to test this idea: calculate observables at higher order for theories with and without explicit pions and compare the qualities of the prediction. In doing such comparisons, however, it is essential to distinguish the quality of the descriptions of the underlying physics from the quality of mere curve fitting. Accordingly, in such comparisons it is essential that the theories with the same number of parameters be compared and the same prescriptions for fitting. Thus, for example the appropriate test for the deuteron form factors of ref. [22] is not whether a higher order effective field theory calculation wit h pions out performs the simple effective range expansion—eventually it must, at least over some region, as one will have additional parameters to characterize the current operator. Rather, the test is whether the theroy with pions out performs an effe ctive field theory with pions integrated out and with the same number of parameters.

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[1] S. Weinberg, *Phys. Lett.* B **251**, 288 (1990); *Nucl. Phys.* B **363**, 3 (1991); *Phys. Lett.* B **295**, 114 (1992).

[2] C. Ordonez and U. van Kolck, *Phys. Lett.* B **291**, 459 (1992); C. Ordonez, L. Ray and U. van Kolck, *Phys. Rev. Lett.* **72**, 1982 (1994) ; *Phys. Rev.* C **53**, 2086 (1996) ; U. van Kolck, *Phys. Rev.* C **49**, 2932 (1994) .

[3] T.S. Park, D.P. Min and M. Rho, *Phys. Rev. Lett.* **74**, 4153 (1995) ; *Nucl. Phys.* A **596**, 515 (1996).

[4] D.B. Kaplan, M.J. Savage and M.B. Wise, *Nucl. Phys.* B **478**, 629 (1996), `nucl-th/9605002`.

[5] T. Cohen, J.L. Friar, G.A. Miller and U. van Kolck, *Phys. Rev.* C **53**, 2661 (1996).

[6] D. B. Kaplan, *Nucl. Phys.* B **494**, 471 (1997).

[7] T.D. Cohen, *Phys. Rev.* C **55**, 67 (1997). D.R. Phillips and T.D. Cohen, *Phys. Lett.* B **390**, 7 (1997). S.R. Beane, T.D. Cohen and D.R. Phillips, *Nucl. Phys.* A **632**, 445 (1998), Ann. Phys. **263** (1998) 255.

[8] K.A. Scaldeferri, D.R. Phillips, C.W. Kao and T.D. Cohen, *Phys. Rev.* C **56**, 679 (1997).

[9] J.L. Friar, Few Body Syst. **99**, 1 (1996), `nucl-th/9607020`.

[10] M.J. Savage, *Phys. Rev.* C **55**, 2185 (1997), `nucl-th/9611022`.

[11] M. Luke and A.V. Manohar, *Phys. Rev.* D **55**, 4129 (1997), `hep-ph/9610534` .

[12] G.P. Lepage, `nucl-th/9706029`, Lectures given at 9th Jorge Andre Swieca Summer School: Particles and Fields, Sao Paulo, Brazil, 16-28 Feb 1997.

[13] S.K. Adhikari and A. Ghosh, J. Phys. **A30**, 6553 (1997).

[14] K. G. Richardson, M. C. Birse and J. A. McGovern, `hep-ph/9708435`; M. C. Birse, J. A. McGovern, and K. G. Richardson `hep-ph/9807302`.

[15] P.F. Bedaque and U. van Kolck, PLB **428**, 221 (1998); P.F. Bedaque, H.-W. Hammer and U. van Kolck, PRC **58**, 641 (1998)

[16] U. van Kolck, Talk given at Workshop on Chiral Dynamics: Theory and Experiment (ChPT 97), Mainz, Germany, 1-5 Sep 1997, `hep-ph/9711222` ; nucl-th/9808007

[17] T. S. Park, K. Kubodera, D. P. Min and M. Rho, `hep-ph/9711463`; T. S. Park `hep-ph/9803417`; T. S. Park, K. Kubodera, D. P. Min and M. Rho, `nucl-th/9807054`

[18] J. Gegelia, `nucl-th/9802038`, `nucl-th/9805008`.

[19] `nucl-th/9806028`.

[20] J.V. Steele and R.J. Furnstahl, NPA **637**, 1998 (16).

[21] D.B. Kaplan, M.J. Savage and M.B. Wise, *Phys. Lett.* B **424**, 390 (1998), `nucl-th/9801034`; `nucl-th/9802075`, *to appear in Nucl. Phys. B*;

[22] D. B. Kaplan, M. J. Savage, and M. B. Wise, `nucl-th/9804032`, *submitted to Phys. Rev. C*

[23] J-W. Chen, H. W. Griesshammer, M. J. Savage, and R. P. Springer `nucl-th/9806080`

[24] T. D. Cohen in Proceedings of "Workshop on Nuclear Physics with Effective Field Theory", to be published.

[25] M. J. Savage, in Proceedings of "Workshop on Nuclear Physics with Effective Field Theory", to be published, `nucl-th/9804034`.

[26] T. D. Cohen and J. M. Hansen, nucl-th/9808006.

[27] T. D. Cohen and J. M. Hansen, nucl-th/9808038.

[28] T. Mehen and I. W. Stewart, nucl-th 9809071.

[29] T. Mehen and I. W. Stewart, 9809095

[30] J. V. Steele and R. J. Furnstahl, nucl-th/9808022.

[31] Reviews of the effective field theory program generally can be found in: A. V. Manohar, hep-ph/9606222; and D. B. Kaplan nucl-th/9506035.

[32] V. G. J. Stoks, R. A.M. Klomp, M. C. M. Rentmeester, and J. J. de Swart, PRC **48**, 792 (1993); on line at http:nn-online.sci.kun.nl

[33] V. G. J. Stoks, R. A.M. Klomp, C. P. F. Terheggen, and J. J. de Swart, PRC **49**, 2950 (1994)

[34] J. J. de Swart, C. P. F. Terheggen, V. G. J. Stoks, nucl-th/9509032, invited talk at "Dubna Deuteron 95".

lab energy (MeV)	$\mathcal{S}$ extracted (Mev)	$\mathcal{S}$ low energy theorem (Mev)
Deuteron Pole	-0.0017 ± 0.0125	-0.743
1	-0.00095 ± 0.00721	-0.0258
5	0.0428 ± 0.0194	-0.535
10	0.245 ± 0.047	-1.78
25	2.18 ± 0.14	-7.54
50	11.03 ± 0.24	-20.10

TABLE I. A comparison of the shape function, $\mathcal{S}(k^2) = k\cot(\delta) + 1/a - 1/2 r_e k^2$ for the $^3S_1$ channel extracted from the Nijmegen partial wave analysis with the prediction by the low energy theorem of eq. (5)

δ ($^3S_1$ channel)	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)
low energy theorem	-.95	4.6	-25.
partial wave analysis	.040	.672	-3.96
Nijm I	.046	.675	-3.97
Nijm II	.045	.673	-3.95
Reid93	0.33	.671	-3.90

TABLE II. A comparison of the effective range expansion coefficients, v_i, for the $^3S_1$ and $^3S_1$ channel predicted from the low energy theorem with coefficients extracted from the partial wave analysis and with three potential models—Nijmegen I, Nijmegen II and Reid 93—which were fit directly to the scattering data.